

Prof. Dr. Alfred Toth

Das System der trajektischen Austauschrelationen

1. Im folgenden wird das System der 81 trajektischen Dualsysteme (vgl. Toth 2025) als duales Abbildungssystem mit Austauschrelationen konstruiert. Selbstduale trajektische Austauschrelationen haben identitive Relationen, markiert durch $\parallel$.

2. Die 81 trajektischen dualen Abbildungssysteme

$$\begin{array}{cc} (1.1, 1.1) & (1.1 \mid 1.1) \\ \parallel & \parallel \\ \times & (1.1 \mid 1.1) \end{array} \quad (1.1, 1.1)$$

$$\begin{array}{cc} (1.1, 1.2) & (1.1 \mid 1.2) \\ \downarrow & \uparrow \\ \times & (2.1 \mid 1.1) \end{array} \quad (2.1, 1.1)$$

$$\begin{array}{cc} (1.1, 1.3) & (1.1 \mid 1.3) \\ \downarrow & \uparrow \\ \times & (3.1 \mid 1.1) \end{array} \quad (3.1, 1.1)$$

$$\begin{array}{cc} (1.1, 2.1) & (1.2 \mid 1.1) \\ \downarrow & \uparrow \\ \times & (1.2, 1.1) \end{array} \quad (1.1 \mid 2.1)$$

$$\begin{array}{cc} (1.1, 2.2) & (1.2 \mid 1.2) \\ \downarrow & \uparrow \\ \times & (2.2, 1.1) \end{array} \quad (2.1 \mid 2.1)$$

$$\begin{array}{ccc}
& (1.1, 2.3) & (1.2 \mid 1.3) \\
& \downarrow \quad \uparrow & \downarrow \quad \downarrow \\
\times & (3.2, 1.1) & (3.1 \mid 2.1)
\end{array}$$

$$\begin{array}{ccc}
& (1.1, 3.1) & (1.3 \mid 1.1) \\
& \downarrow \quad \uparrow & \uparrow \quad \downarrow \\
\times & (1.3, 1.1) & (1.1 \mid 3.1)
\end{array}$$

$$\begin{array}{ccc}
& (1.1, 3.2) & (1.3 \mid 1.2) \\
& \downarrow \quad \uparrow & \downarrow \quad \downarrow \\
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$$\begin{array}{ccc}
& (1.2, 1.1) & (1.1 \mid 2.1) \\
& \uparrow \quad \downarrow & \downarrow \quad \uparrow \\
\times & (1.1, 2.1) & (1.2 \mid 1.1)
\end{array}$$

$$\begin{array}{ccc}
& (1.2, 1.2) & (1.1 \mid 2.2) \\
& \downarrow \quad \downarrow & \downarrow \quad \uparrow \\
\times & (2.1, 2.1) & (2.2 \mid 1.1)
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\times & (1.1, 3.1) & (1.3 \mid 1.1)
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\times & (2.1, 3.1) & (2.3 \mid 1.1)
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$$\begin{array}{ccc}
& (1.3, 3.1) & (1.3 \mid 3.1) \\
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& (2.1, 1.1) & (2.1 \mid 1.1) \\
& \uparrow \quad \downarrow & \uparrow \quad \downarrow \\
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 \end{array}$$

Literatur

Toth, Alfred, Trajektische Dualrelationen. In: Electronic Journal for Mathematical Semiotics, 2025

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